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Technological progress, income inequality, and fertility

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Abstract This paper constructs an overlapping-generations model with two different types of technology: modern, which can be accessed only by the skilled, and traditional, which can be accessed by the unskilled. The model described in this paper shows that a rise in the wage premium for skilled workers caused by skill-biased technological changes explains the following key stylized facts: with economic development, the fraction of skilled people increases, the fertility rate declines, and income inequality rises and then falls. The model also explains the observed gradual rises in income inequality in developed countries.

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1 Introduction

Long-run historical data shows several key features in the process of economic development: increases in the fraction of people who receive education, declines in the fertility rate, and rises followed by declines in income inequality (the so-called Kuznets Curve). For example, in the United Kingdom, the average years of schooling increased from 2 years in 1820 to 8 years in 1913 and reached 14.1 years in 1992 (see, for a review, Maddison 1995). The total fertility rate declined from 4.94 in 1875 to 2.4 in 1920 and reached around 1.5 in 1990 (see, for a review, Chesnais 1992). Income inequality (wealth inequality) reached its peak around 1870 and declined thereafter (see, for a review, Williamson 1985).

Many studies, including those written by Galor and Weil (2000) and Hansen and Prescott (2002), have recently tried to explain key features of economic development in a unified model. Among them, to explain the three observed facts stated above, Dahan and Tsiddon (1998), Morand (1999), Kremer and Chen (2002), de la Croix and Doepke (2003, 2004), and Doepke (2004) constructed models with endogenous fertility and explicitly discussed the evolution of human capital investment, fertility rate, and income inequality.

Their analyses are as follows: parents with a lower (higher) level of human capital decide to have more (fewer) children and invest less (more) in education. Due to this difference in the fertility rate among parents, children who receive less (more) education make up a larger (smaller) fraction of the population in the next period. If there is a sufficient degree of inter-generational persistence, children of parents with lower (higher) human capital also decide to have more (fewer) children and invest less (more) in education. At this phase, the population's overall level of education remains constant, the fertility rate increases, and income becomes more unequally distributed. For the transition from this stagnation phase to growth to occur, exogenous or endogenous changes which increase the returns from education must be considered. Morand (1999) emphasized the role of the Lucas (1988)-type external effect of human capital investment. Dahan and Tsiddon (1998) and Kremer and Chen (2002) focused upon the role of increases in the wage premium for skilled workers caused by changes in the labor force composition of skilled and unskilled workers. de la Croix and Doepke (2003, 2004) stressed the roles of both exogenous technological change and the Lucas (1988)-type external effect of human capital investment. Once these changes occur, even dynasties, who initially invested less in education, come to find it profitable to invest more, and an economy starts to grow. Through this process, the population's overall level of education rises, fertility declines, and income becomes more equally distributed.

The analyses given above are appealing and plausible. However, there may exist other plausible analyses that explain these historically observed facts well. Moreover, as most of these existing models possess a relatively complicated structure and mechanism, it is difficult to understand analytically what specifications or assumptions of their model are keys to derive their results. In this sense, these existing models are not very tractable. The purpose of this paper is to construct a tractable growth model that explains these observed facts in a different mechanism from those described in existing studies. In our model, a unique factor, that is, rises in the skilled wage premium due to skill-biased technological changes, plays a key role in explaining the historically observed patterns of the skilled (educated) agent fraction, the total fertility rate, and income inequality.

The model described in this paper has two distinct features. First, we consider two different types of technology: modern, which can be accessed only by the skilled, and traditional, which can be accessed by the unskilled.¹ Then, the advancement in the productivity of modern technology is assumed to be higher than that of traditional technology. This assumption is supported by empirical studies. For example, using the data on the United Kingdom in the period from 1780 to 1860, Harley (1996) showed that the productivity growth rate in the modernized sector (the modern technology) was 1.2%, whereas that in the agricultural sector (the traditional technology) was 0.7%. Due to this skill-biased technological change, the skilled wage premium increases with economic growth.

Second, we explicitly consider the complementarity between human capital accumulation and technological change. The assumption that the current level of productivity is related to some measure of past educational level has been widely used in theoretical analyses (e.g., Buiter and Kletzer 1993; Galor and Tsiddon 1997; Mountford 1997) and is empirically supported at both the macro (e.g., Barro 1991; Mankiw et al. 1992) and the micro levels (e.g., Becker and Tomes 1996). Due to this complementarity, an economy can begin the transition from stagnation to growth if a once-and-for-all exogenous shock induces a significant fraction of people to be educated at some point in time.²

The analysis shows that our model has three types of equilibria: a ‘poverty trap equilibrium,’ characterized by no technological progress, a ‘partially skilled equilibrium,’ characterized by positive technological progress and the coexistence of skilled and unskilled agents, and a ‘fully skilled equilibrium,’ characterized by positive technological progress and the non-existence of unskilled agents. If the initial level of technology or of the skilled agent fraction is too low to induce technological progress, the economy is in a poverty trap equilibrium.³ On the other hand, if the initial levels of technology and of the skilled agent fraction are sufficiently high, technological progress occurs, and the economy eventually reaches a partially skilled equilibrium or a fully skilled equilibrium. Whether the economy reaches the former or the latter depends upon the extent to which technological progress improves labor productivity.

Let us consider an economy that is initially in the poverty trap equilibrium. In this case, we consider two examples that enable the economy to escape from the poverty trap and eventually reach the fully skilled equilibrium. The first example stresses the role of an exogenous increase in longevity of agents that is caused by, for example, improvements in public health infrastructure, including systems for waste disposal, clean water supply, drainage and sewage, and basic health care. The second example emphasizes the role of an exogenous decrease in the educational costs, which is caused by, for example, a decline in tuition fees due to the introduction of public education. An increase in longevity raises the future returns from being skilled, whereas a decrease in educational costs lowers the costs of obtaining skills. Therefore, either of these two exogenous changes induces more agents to receive education, increases the fraction of skilled agents, and, thus,

¹ To the best of our knowledge, few existing studies have examined the evolution of income inequality and fertility using a model with two types of technology.

² This is in contrast to a study by Hansen and Prescott (2002), in which sustained growth is the result of continuously improving exogenous technology.

³ Galor (2004) denotes this type of equilibrium as the Malthusian state.

enhances the technological progress due to the complementarity between human capital accumulation and technological change.

During the transition process caused by either of these two exogenous changes, the skilled wage premium increases due to skill-biased technological changes. Hence, the skilled agent fraction increases monotonously, and all agents eventually become skilled. A rise in the skilled wage premium reduces the total fertility rate in the following two ways. First, it decreases the fraction of unskilled agents who are likely to have many children. Second, it directly reduces the fertility rate of skilled agents, as it increases their opportunity cost of childbearing. Therefore, the total fertility rate decreases monotonously. Moreover, a rise in the skilled wage premium influences income inequality in the following two ways. On the one hand, it raises income inequality between skilled and unskilled agents, an effect that we denote as the “skill premium effect.” On the other hand, it increases the relative number of skilled agents and thus increases the fraction of agents who can reap the benefits of technological progress, resulting in a decline in income inequality as a whole. We denote this effect as the ‘skilled cohort effect.’ At low technological levels, as the skilled agent fraction is still small, the ‘skill premium effect’ dominates the ‘skilled cohort effect,’ and income inequality rises.⁴ However, at high technological levels, as the skilled agent fraction is sufficiently large, the ‘skilled cohort effect’ dominates the ‘skill premium effect,’ and income inequality declines. Therefore, income inequality first rises and then declines with technological advancement.

Thus, our model can replicate all of the historically observed patterns of the skilled agent fraction, the total fertility rate, and income inequality, with emphasis upon the role of skill-biased technological change. In addition to these main theoretical results, we also show that, if all agents become skilled, income inequality begins to increase again at a very slow speed because income inequality among skilled agents rises with the advancement of technology. This is consistent with recent empirical results described in Atkinson (1995), which demonstrated that income inequality in developed countries such as the United States of America and the United Kingdom increased during the 1980s and 1990s.

This paper is organized as follows. Section 2 establishes the basic structure of the model. Section 3 presents the condition of three types of equilibria. Section 4 shows the evolution of the skilled agent fraction, the fertility rate, and income inequality. It also discusses the relationship between income inequality and the fertility differential. Section 5 presents the conclusion.

2 The model

Let us consider an economy with an infinite number of periods and overlapping generations of agents who live for a maximum of three periods. There is only one kind of good, and it is treated as a numeraire. The economy begins its operation in period 0, and a cohort born in period t is called generation t . The first, second, and third periods of life of agents are referred to as the young, middle-aged, and old periods, respectively. Agents in this model are born as young and survive through

⁴ The existing studies mainly focused upon differences in the level of education and fertility rate among dynasties as a cause for the rise in income inequality. This paper only focuses upon the role of the rise in the wage premium for skilled workers, which greatly simplifies our analysis.

their middle age. A middle-aged agent has a probability p of surviving to his/her old age.

For analytical tractability, we assume that agents are members of single-worker firms when they are young and middle-aged.⁵ We consider two types of agents: skilled agents who received education when young and unskilled agents who did not. We also consider two different types of technology: modern, which can be accessed only by the skilled workers, and traditional, which can be accessed by the unskilled workers. The advancement in the productivity of modern technology is assumed to be higher than that in traditional technology.

There is a continuum of agents, characterized by the innate potential parameter h , which is the level of human capital achieved by receiving education. The cumulative distribution function of the innate potential is denoted by $F(h)$, with the support being the interval $[h_{min}, h_{max}]$. The density function is denoted by $f(h)$. Young agents work inelastically as unskilled workers and choose whether to receive education.⁶ In this paper, it is assumed that agents can receive education by paying a fixed amount of tuition, e . However, the level of human capital, h , which each agent can gain depends upon his/her innate potential.⁷

If young agents receive education, they can access modern technology and work as skilled agents in their middle age. The modern technology of a single-worker firm in this model is defined as follows:

$$Y_{t+1}^M = \psi(A_{t+1})L_{t+1}^M, \quad (1)$$

where A_{t+1} represents the level of technology in period $t + 1$, L_{t+1}^M is the labor input in terms of efficiency units, and $\psi(\cdot)$ is a function that governs the feature of the relationship between the level of technology and the productivity of labor in terms of efficiency units in modern technology. We assume that $\psi'(\cdot) > 0$, $\psi''(\cdot) < 0$, $\psi(0) = \underline{\psi} > 0$, and $\lim_{A_t \rightarrow \infty} \psi(A_t) = \bar{\psi} > 0$. Thus, the higher the value of A_{t+1} , the higher the level of labor productivity. In addition, the upper shift of the function $\psi(\cdot)$ and the rise in the value of $\bar{\psi}$ imply that technological progress improves labor productivity. It is noteworthy that the output Y_{t+1}^M of each single-worker firm is identical to the income of its agent, as the product is treated as a numeraire.

If young agents do not receive education, they have to access traditional technology and work as unskilled laborers in both their young and middle-aged periods, during which they are endowed with one unit of (raw) labor. The traditional technology of a single-worker firm in this model is defined as follows:

$$Y_{t+1}^T = w(B_{t+1})L_{t+1}^T, \quad (2)$$

where B_t represents the level of technology in period $t + 1$ and L_{t+1}^T is the labor input (in terms of raw labor). $w(\cdot)$ is a function that governs the feature of the relationship between the level of technology and the productivity of labor in the traditional technology. In this case, the output Y_{t+1}^T of each single-worker firm is again identical to the income of its agent.

⁵ Existing studies that adopted this assumption of single-worker firms include those by Blackburn and Cipriani (2002) and Bertinelli and Black (2004).

⁶ We assume that all agents have the basic knowledge necessary to work as unskilled agents before they enter the economy.

⁷ This type of specification for individual education is common in the literature. See Razin et al. (2002), for example.

Following Laitner (2000) and Galor and Mountford (2003), it is assumed in the present study that the rate of technological progress in the modern sector is higher than that in traditional technology. To describe this property in a simple way, we assume that the level of technology in the traditional sector B_t is constant over time. That is, $B_t = B$ for all t .⁸ Hereafter, for notational simplicity, we denote the level of labor productivity in the traditional technology $w(B)$ as w (i.e., $w = w(B)$). Therefore, we only explicitly discuss the advancement in the productivity of the modern sector. As the level of technology A_t is irreversible, as is common in the literature, A_t satisfies the following condition, $A_{t+1} \geq A_t \geq A_0$ for all t , where A_0 is the level of technology in the initial period. The property of technological progress is discussed more rigorously later.

Agents derive utility from their own consumption while middle-aged and old and from the number of children they have. Thus, the expected lifetime utility of the agent i in generation t , whose innate potential is h_i , is expressed as:

$$V_{i,t+1} = \alpha \ln(c_{i,t+1}) + \beta \ln(n_{i,t+1}) + p\gamma \ln(d_{i,t+2}), \quad (3)$$

where $c_{i,t+1}$ and $d_{i,t+2}$ refer to consumption while middle-aged and old and $n_{i,t+1}$ refers to the number of children. In this case, α , γ , and β measure the agent's taste for consumption during middle age and old age, and the taste for the number of children.

In the first (young) period of life, all agents are endowed with one unit of labor, work inelastically as unskilled workers, and obtain income w by accessing traditional technology. Agents also decide whether to invest e to become skilled or save all their income in their young period using storage technology. We assume that agents can finance the educational cost e without borrowing from the financial market (i.e., $e < w$).⁹

A person i who invests e is denoted as 'skilled,' whereas a person i who does not invest e is denoted as 'unskilled.' In the second (middle-aged) period of life, agents work as skilled or unskilled agents and decide on the number of children to have. They also divide their income between consumption $c_{i,t+1}$ and saving $s_{i,t+1}$ for their old-age period. We assume that insurance companies are risk-neutral and that the private annuities market is competitive. Insurance companies promise agents a payment, $(R/p)b_{i,t+1}$, in exchange for which the estate $b_{i,t+1}$ accrues to the companies, where p is the average survival probability and R is the gross rate of the return of storage technology.¹⁰ In the absence of a bequest motive, agents are willing to invest their assets in such insurance. Finally, in the third (old) period of life, survivors retire and spend their remaining income on consumption $d_{i,t+2}$.

⁸ This assumption is made for simplicity. Even if we do not make this assumption, we can still obtain analogous theoretical results when we assume that the advancement in the productivity of traditional technology is always lower than that in modern technology. However, the analysis becomes complicated without resulting in a qualitative change in the results.

⁹ As the issue of financial market imperfection is not considered in this study, only the case in which the relation $e < w$ holds is examined.

¹⁰ The company sells annuities to young agents and invests the proceeds in real investments. In the next period, the returns on the investment are repaid to the insured old agents, who are still living. Thus, the rate of the returns on the annuities in period $t + 1$ is R/p . For example, see Blanchard (1985) for more details.

Thus, the budget and time constraints of a skilled agent denoted by superscript s with innate potential h_i are:

$$c_{i,t+1}^s + b_{i,t+1}^s = \psi(A_{t+1})h_i I_{i,t+1}^s + R(w - e), \quad (4)$$

$$d_{i,t+2}^s = \frac{R}{p} b_{i,t+1}^s, \quad (5)$$

$$l_{i,t+1}^s + \tau n_{i,t+1}^s = 1, \quad (6)$$

where $c_{i,t+1}^s$, $d_{i,t+2}^s$, $b_{i,t+1}^s$, $l_{i,t+1}^s$, and $n_{i,t+1}^s$ refer to middle-age consumption, old-age consumption, middle-age saving, middle-age labor supply, and the number of children of a skilled worker with innate potential h_i , respectively. Following Becker (1965) and others, we assume that it takes a fixed amount of time τ to bear and raise each child.

By maximizing Eq. (3), subject to Eqs. (4), (5), and (6), we obtain

$$c_{i,t+1}^s = \frac{\alpha}{\alpha + \beta + \gamma p} I_{i,t+1}^s, \quad (7)$$

$$d_{i,t+2}^s = \frac{\gamma R}{\alpha + \beta + \gamma p} I_{i,t+1}^s, \quad (8)$$

$$n_{i,t+1}^s = \frac{\beta}{\tau(\alpha + \beta + \gamma p)} \frac{I_{i,t+1}^s}{\psi(A_{t+1})h_i}, \quad (9)$$

where $I_{i,t+1}^s$ represents the potentially disposable income of a skilled agent and is defined as

$$I_{i,t+1}^s \equiv \psi(A_{t+1})h_i + R(w - e).$$

It is the disposable income of a skilled agent with innate potential h_i who spends all his/her time on working and no time on child bearing. The indirect utility function is then

$$v_{i,t+1}^s = X (I_{i,t+1}^s)^{\alpha + \beta + \gamma p} [\psi(A_{t+1})h_i]^{-\beta}, \quad (10)$$

where

$$X \equiv \left(\frac{\alpha}{\alpha + \beta + \gamma p} \right)^\alpha \left[\frac{\beta}{\tau(\alpha + \beta + \gamma p)} \right]^\beta \left(\frac{\gamma R}{\alpha + \beta + \gamma p} \right)^{\gamma p}.$$

We can see from Eq. (9) that a rise in the level of technology A_{t+1} lowers the demand for children of skilled agents. This is because a rise in A_{t+1} enhances the labor productivity of skilled agents and increases their opportunity cost of having children.

On the other hand, the budget and time constraints of an unskilled agent denoted by superscript u with innate potential h_i are:

$$c_{i,t+1}^u + b_{i,t+1}^u = w l_{i,t+1}^u + R w, \quad (11)$$

$$d_{i,t+2}^u = \frac{R}{p} b_{i,t+1}^u, \quad (12)$$

$$l_{i,t+1}^u + \tau n_{i,t+1}^u = 1, \quad (13)$$

where $c_{i,t+1}^u$, $d_{i,t+2}^u$, $b_{i,t+1}^u$, $l_{i,t+1}^u$, and $n_{i,t+1}^u$ are middle-age consumption, old-age consumption, middle-age saving, middle-age labor supply, and the number of children of an unskilled worker with innate potential h_i , respectively.

By maximizing Eq. (3), subject to Eqs. (11), (12), and (13), we obtain

$$c^u = c_{i,t+1}^u = \frac{\alpha}{\alpha + \beta + \gamma p} I^u, \quad (14)$$

$$d^u = d_{i,t+2}^u = \frac{\gamma R}{\alpha + \beta + \gamma p} I^u, \quad (15)$$

$$n^u = n_{i,t+1}^u = \frac{\beta}{\tau(\alpha + \beta + \gamma p)} \frac{I^u}{w}, \quad (16)$$

where I^u represents the potentially disposable income of all unskilled workers and is defined as

$$I^u \equiv (1 + R)w.$$

This is considered to be disposable income if an unskilled worker spends all his/her time working. The indirect utility function is then

$$v^u = X(I^u)^{\alpha+\beta+\gamma p} w^{-\beta}. \quad (17)$$

As the labor productivity level of traditional technology w is assumed to be constant and all unskilled agents are endowed with the same one unit of labor, they obtain the same potentially disposable income I^u and the same lifetime utility level v^u , which are constant over time. In addition, from Eq. (16), the level of technology A_{t+1} does not have any impact upon the unskilled agent's demand for children simply because unskilled agents cannot access modern technology.

For the sake of clarity in the following discussion, we restrict our analysis to a set of parameters satisfying the following condition and denote it as Assumption 1.

Assumption 1

$$\psi(A_0)h_{min} + R(w - e) > (1 + R)w.$$

Assumption 1 implies that the potentially disposable income of a skilled worker in the initial period is larger than that of an unskilled worker, even for the agent

with the lowest innate potential. Given Assumption 1, with regard to the fertility behavior of the skilled and unskilled agents, we have the following lemma.

Lemma 1 *The following inequality holds for any agent with innate potential h_i :*

$$n_{i,t+1}^s < n^u.$$

The proof of Lemma 1 is given in “Appendix A.” Lemma 1 indicates that an agent with innate potential h_i has more children when he becomes unskilled than when he becomes skilled because the opportunity cost of childbearing is higher for a skilled agent than for an unskilled one. From Eq. (6), we can see that the fertility rate of an unskilled agent is always constant at n^u for any agent’s innate potential h_i , and, hence, the average fertility rate of unskilled agents is also constant at n^u . Thus, Lemma 1 implies that the average fertility rate of skilled agents is always lower than that of unskilled agents.

Next, we consider the educational choice of an agent with innate potential h_i . If $v_{i,t+1}^s \geq v^u$, the agent receives education and becomes skilled.¹¹ From Eqs. (10) and (17), we have the following relationship:

$$v_{i,t+1}^s \geq v^u \Leftrightarrow \frac{[z_{t+1} + R(w - e)]^{\alpha+\beta+\gamma p}}{z_{t+1}^\beta} \geq (1 + R)^{\alpha+\beta+\gamma p} w^{\alpha+\gamma p}, \quad (18)$$

where $z_{t+1} \equiv \psi(A_{t+1})h_i$.

To clarify the discussion, in the following analysis, we restrict our analysis to the set of parameters satisfying the following condition and denote it as Assumption 2.

Assumption 2

$$\frac{[\underline{z} + R(w - e)]^{\alpha+\beta+\gamma p}}{\underline{z}^\beta} < (1 + R)^{\alpha+\beta+\gamma p} w^{\alpha+\gamma p},$$

where \underline{z} is defined as $\underline{z} = \psi(A_0)h_{min}$.

Assumption 2 implies that the agent with the lowest innate potential becomes unskilled in the initial period. Given Assumption 2, as rigorously shown in “Appendix B,” there exists a unique value of z^* , which satisfies Eq. (18) with equality:

$$\frac{[z^* + R(w - e)]^{\alpha+\beta+\gamma p}}{z^{*\beta}} = (1 + R)^{\alpha+\beta+\gamma p} w^{\alpha+\gamma p}.$$

¹¹ For expositional simplicity, we assume that agents choose to receive education if it is indifferent to them whether they become skilled or unskilled.

3 Equilibrium

Given the feature of technological progress and the two lemmas discussed above, we can examine the equilibrium of our model. We focus on the following three types of equilibria: a ‘poverty trap equilibrium,’ characterized by no technological progress, a ‘partially skilled equilibrium,’ characterized by positive technological progress and the coexistence of skilled and unskilled agents, and a ‘fully skilled equilibrium,’ characterized by positive technological progress and the non-existence of unskilled agents. Once the economy goes into one of these equilibria, it stays in it permanently.

In the remaining part of this section, we investigate the conditions under which the above three types of equilibria are attained. First, we derive conditions under which the economy is in the poverty trap equilibrium. From Lemma 2, if the initial level of technology A_0 is sufficiently low to satisfy the condition that

$$h_{max} < \frac{z^*(p, e)}{\psi(A_0)}, \quad (20)$$

no agents become skilled in the initial period. In this case, from Eq. (19), as $s_0 = 0$ and $g(0) < 1$, technological progress does not occur in the initial period. As the level of technology remains at initial level A_0 , none of the agents in the next period is willing to become skilled. As a consequence, from Eq. (19), the level of technology again remains at A_0 in the next period and all subsequent periods.

Even when the initial level of technology A_0 is high enough to violate Eq. (20) and some agents become skilled workers, if the skilled worker fraction in the initial period s_0 is sufficiently low to satisfy the condition that

$$g(s_0) \leq 1, \quad (21)$$

technological progress does not occur. As the level of technology remains at the initial level A_0 , the skilled worker fraction in the next period remains constant at the initial level s_0 . As a consequence, again from Eq. (19), the level of technology permanently remains at A_0 .

Summarizing the above arguments, we obtain the following proposition.

Proposition 1 *If the economy satisfies one of the following two sets of conditions, the economy is in the poverty trap equilibrium.*

- $h_{max} < \frac{z^*(p, e)}{\psi(A_0)}$ and $g(0) \leq 1$.
- $h_{max} \geq \frac{z^*(p, e)}{\psi(A_0)}$ and $g(s_0) \leq 1$.

Proposition 1 indicates that, if the initial level of technology A_0 or the initial skilled agent fraction s_0 is low enough to satisfy Eq. (20) or Eq. (21), technological progress does not occur, implying that the economy is in the poverty trap equilibrium.

We next derive the condition under which the economy attains the partially skilled equilibrium. From the argument concerning Proposition 1, if the initial

level of technology A_0 and the skilled agent fraction s_0 are sufficiently high to satisfy the condition that

$$h_{max} \geq \frac{z^*(p, e)}{\psi(A_0)} \quad \text{and} \quad g(s_0) > 1, \quad (22)$$

technological progress occurs in the initial period. This rise in the level of technology A_t , from Lemma 2, increases the skilled agent fraction s_t in the next period, which again, from Eq. (19), induces technological progress in subsequent periods. Thus, we can find a positive-feedback mechanism between the rate of technological progress and the skilled agent fraction. Note that the level of technology is irreversible and satisfies the condition that $A_{t+1} \geq A_t \geq A_0$ for all t . From Lemma 2, this property implies that s_t also satisfies the condition that $s_{t+1} \geq s_t \geq s_0$ for all t . Therefore, if s_0 is sufficiently high to induce technological progress, so is s_t in all subsequent periods. Due to this mechanism, under the condition of Eq. (22), A_t increases monotonously and approaches infinity in the long run. In other words, concerning the labor productivity of modern technology, we can find the following relationship: $\lim_{t \rightarrow \infty} \psi(A_t) = \lim_{A_t \rightarrow \infty} \psi(A_t) = \bar{\psi}$. However, even when A_t approaches infinity in the long run, the maximum level of labor productivity $\bar{\psi}$ may be low enough to satisfy the following condition:

$$h_{min} < \frac{z^*(p, e)}{\bar{\psi}}. \quad (23)$$

If this condition holds, there always exist some agents who remain unskilled. Under Eqs. (22) and (23), there always exist both skilled and unskilled agents.

The above arguments imply the following proposition.

Proposition 2 *The economy is in the partially skilled equilibrium if the following set of conditions is satisfied.*

$$- h_{max} \geq \frac{z^*(p, e)}{\psi(A_0)}, g(s_0) > 1 \text{ and } h_{min} < \frac{z^*(p, e)}{\bar{\psi}}$$

Proposition 2 indicates that, if the initial level of technology A_0 and the skilled agent fraction s_0 are sufficiently high to satisfy Eq. (22), the technological level begins to rise. This makes the skilled agent fraction increase, which again induces technological progress due to the complementarity between human capital accumulation and technological progress. In this way, the economy grows. However, if technological progress improves the labor productivity of agents less effectively so that Eq. (23) is satisfied, there remain some unskilled agents.

Finally, we derive a condition under which the economy attains the fully skilled equilibrium. From the argument concerning Propositions 1 and 2, if Eq. (22) holds and the maximum level of labor productivity $\bar{\psi}$ is sufficiently high to satisfy the condition that

$$h_{min} > \frac{z^*(p, e)}{\bar{\psi}}, \quad (24)$$

the level of technology increases monotonously, and all agents become skilled in the long run.

In this case, the following proposition holds.

Proposition 3 *The economy is eventually in the fully skilled equilibrium if the following set of conditions is satisfied.*

$$- h_{max} \geq \frac{z^*(p,e)}{\psi(A_0)}, g(s_0) > 1 \text{ and } h_{min} \geq \frac{z^*(p,e)}{\psi}.$$

Proposition 3 indicates that, if technological progress improves the labor productivity sufficiently so that Eq. (24) is satisfied, all agents become skilled in the long run.

From Propositions 1 to 3, it is clear that, if the initial level of technology A_0 or the initial skilled agent fraction s_0 is sufficiently low, the economy is in the poverty trap equilibrium. If A_0 and s_0 are high, the economy is eventually in the partially skilled equilibrium or in the fully skilled equilibrium. Hence, our model has two sets of multiple equilibria: one consists of the poverty trap equilibrium and the partially skilled equilibrium, and the other consists of the poverty trap equilibrium and the fully skilled equilibrium. Whether the economy has the former or the latter type depends upon the extent to which technological progress improves labor productivity.

4 The skilled agent fraction, fertility rate, income inequality, and technological progress

In this section, we present two examples that demonstrate that our model can replicate some of the key development facts mentioned in the “Section 1:” increases in the skilled (educated) agent fraction, declines in fertility, and the Kuznets Curve.¹³

Let us consider an economy whose initial level of technology A_0 and skilled agent fraction s_0 satisfy the conditions $h_{max} \geq z^*(p, e)/\psi(A_0)$ and $g(s_0) \leq 1$. From Proposition 1, in this case, the economy is stuck in a low-skill, no technological-growth poverty trap.

The first example stresses the role of *the exogenous increases in the longevity of agents* as an important driving force which enables the economy to escape from the poverty trap. Increases in the longevity of agents are considered to be caused by, for example, improvements in the public health infrastructure (e.g., systems for waste disposal, clean water supply, drainage and sewage, and basic health care). According to Steckel and Floud (1997), the germ theory of diseases came to be widely accepted in the medical profession by the 1880s. The application of such knowledge in the form of public health policies, such as those improving waste disposal, clean water supply, and antiseptic medical procedures, was ultimately

¹³ While we do not discuss it explicitly, an exogenous increase in the initial level of technology A_0 caused by the knowledge spillover from abroad also increases the initial skilled agent fraction s_0 , violates Eq. (19), and enables the economy to escape from the poverty trap. Gerschenkron (1962) stated that it is important for developing countries to absorb technologies from developed countries. In fact, some countries, such as Japan, Korea, and Singapore, absorbed technologies from European countries and the USA in their process of early development.

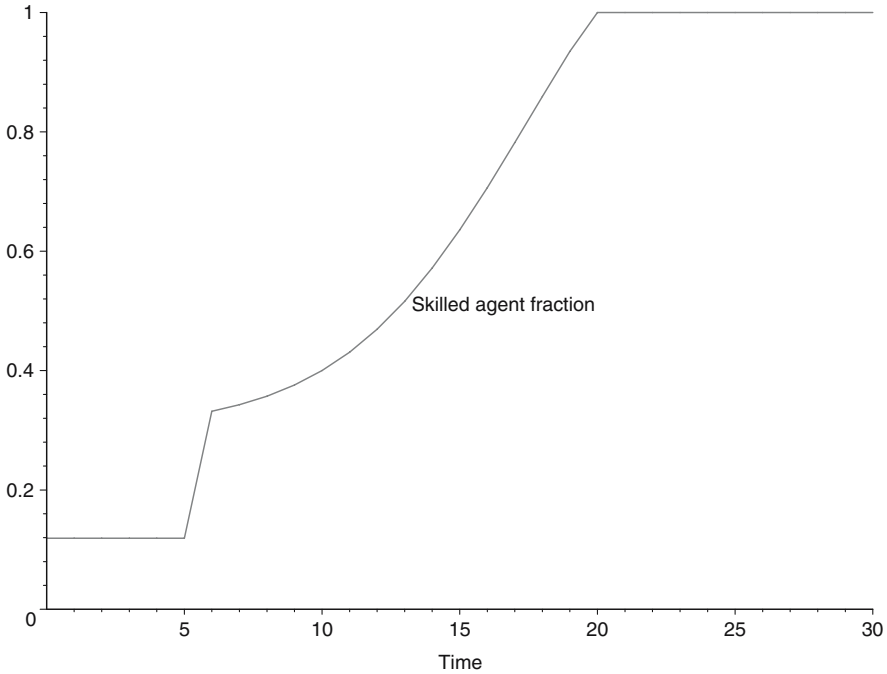


Fig. 1 The evolution of skilled agent fraction. The survival probability p increases from 0.4 to 0.7 at period 5

effective in enhancing the longevity of agents in European countries during the late 19th and early 20th centuries. Now, let us consider an exogenous rise in the survival probability p .¹⁴ As a rise in p lowers z^* because $dz^*/dp < 0$, it increases the number of agents who become skilled. If the rise in p is large enough to make s_0 violate Eq. (21), technological progress occurs. Then, the economy escapes from the poverty trap.

The second example stresses the role of *the exogenous decreases in the cost of education*. According to Galor (2004), the introduction of public education, which lowered the educational costs, generated a significant increase in the supply of educated workers in the second half of the 19th century. For example, in England, the proportion of children aged 5 to 14 in primary schools increased from 11% in 1855 to 74% in 1900. As shown in Flora et al. (1983), a similar pattern is observed in other European countries. In particular, in France, the percentage increased from 30% in 1832 to 86% in 1901. Now, let us consider an exogenous decrease in the educational cost e . As a decrease in e lowers z^* because $dz^*/de > 0$, it increases the number of agents who become skilled. If the decrease in e is large enough to make s_0 violate Eq. (21), technological progress occurs. Then, the economy escapes from the poverty trap.

To explain the evolution of the skilled agent fraction, the total fertility rate, and income inequality during the transition process caused by the exogenous rise in

¹⁴ Various existing studies investigate the impact of the exogenous rise in longevity upon economic development. See Kalemli-Ozcan et al. (2000), for example.

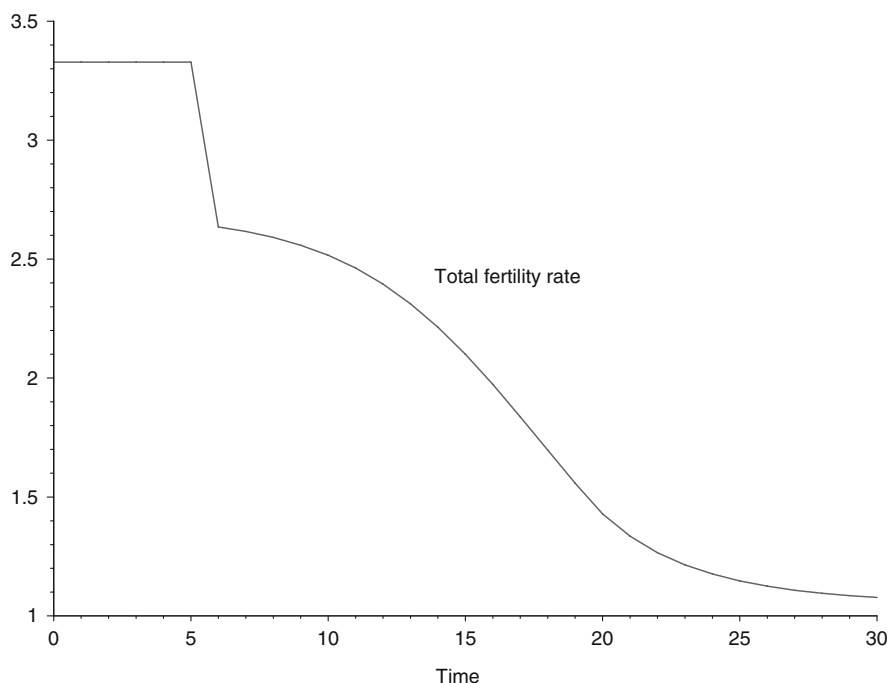


Fig. 2 The evolution of total fertility rate. The survival probability p increases from 0.4 to 0.7 at period 5

p and the decrease in e more concretely, we present numerical simulation results. The functions and parameters used in the baseline simulations are given in “Appendix C”.¹⁵ In the baseline simulations, the economy is in a poverty trap equilibrium characterized by the conditions $h_{max} \geq z^*(p, e)/\psi(A_0)$ and $g(s_0) \leq 1$ in the initial period. Then, in period 5, exogenous shocks occur. In the first example, the survival probability p increases from 0.4 to 0.7. In the second example, the educational cost e decreases from 3 to 2. Due to the rise in p and the decrease in e , the economy escapes from the poverty trap, begins to grow, and gradually converges to a fully skilled equilibrium. Figures 1, 2 and 3 show the results when p increases, whereas Figs. 4, 5, and 6 show the results when e decreases. For clarity of explanation, we mainly discuss the results of Figs. 1, 2, and 3. We can obtain analogous intuitions from the results of Figs. 4, 5 and 6.

Figures 1, 2 and 3 present the evolution of the skilled agent fraction (Fig. 1), total fertility rate (Fig. 2), and income inequality measured by the Theil index (Fig. 3) during the transition process caused by the exogenous rise in p .¹⁶ As the initial level of the technology A_0 is high enough to violate Eq. (20), some agents become skilled in the initial period. However, as shown in Fig. 1, the initial skilled agent fraction is sufficiently low to satisfy Eq. (1). Therefore, technological progress does not occur, and the skilled agent fraction s_t remains constant at the initial level s_0

¹⁵ Rigorous sensitivity analyses were undertaken for different combinations of the parameter values, and the findings of the paper hold for a wide range of values.

¹⁶ For details on the Theil index, see Theil (1979), for example.

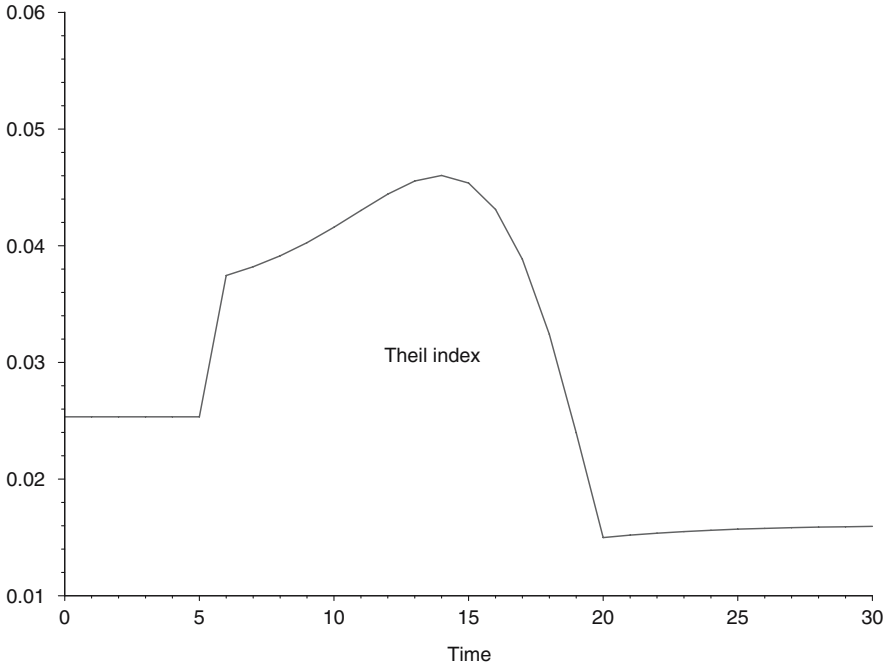


Fig. 3 The evolution of income inequality measured by Theil index. The survival probability p increases from 0.4 to 0.7 at period 5

in the poverty trap equilibrium. This result implies that the fraction of unskilled agents who are likely to have many children is high and constant. In addition, from Eqs. (9) and (15), the average fertility rate of both skilled and unskilled agents remains constant. Therefore, as shown in Fig. 2, the total fertility rate is high and constant over time in the poverty trap equilibrium. Due to the coexistence of skilled and unskilled agents, there already exists income inequality among agents in the initial period. However, as technological progress does not occur, the skill premium remains constant at the initially low level. Therefore, as shown in Fig. 3, income inequality among agents, which is mainly determined by the distribution of innate potential, is relatively small and remains constant in the poverty trap equilibrium.

The exogenous increase in the survival probability p in period 5 breaks through the constraints of Eq. (21) and induces technological progress. This rise in the level of technology A_t increases the wage premium for skilled workers. Therefore, from Lemma 2, it increases the skilled agent fraction s_t in the next period, which, again, from Eq. (19), induces technological progress in subsequent periods. Due to this positive feedback mechanism between the rate of technological progress and the skilled agent fraction, the skilled agent fraction increases monotonically, all agents eventually become skilled, and the economy reaches the fully skilled equilibrium. In our simulation, as shown in Fig. 1, all agents become skilled in period 20.

The rise in the skilled wage premium reduces the total fertility rate in two different ways. First, it decreases the fraction of unskilled agents who are likely to have more children (see Lemma 1). Second, it reduces the fertility rate of skilled agents

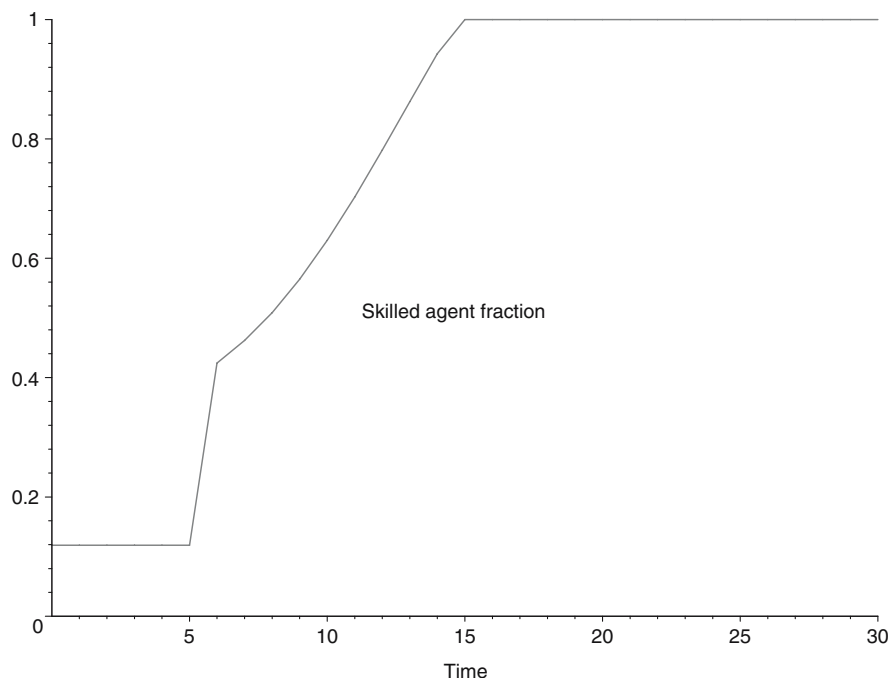


Fig. 4 The evolution of skilled agent fraction. The educational e decreases from 3 to 2 at period 5

because it increases their opportunity cost of childbearing (see Eq. 9). Therefore, as shown in Fig. 2, the total fertility rate of this economy decreases monotonically.

Moreover, the rise in the skilled wage premium influences the income inequality in two different ways. On the one hand, it increases the income inequality between skilled and unskilled agents, an effect that we denote as the ‘skill premium effect,’ which positively affects income inequality among agents. On the other hand, it increases the skilled agent fraction s_t and thus the share of agents who can reap the benefits of technological progress. We denote this effect as the ‘skilled cohort effect,’ which negatively affects income inequality among agents. In our simulation, as shown in Fig. 3, from periods 5 to 14, the skilled agent fraction is still sufficiently low. Therefore, the ‘skill premium effect’ dominates the ‘skilled cohort effect,’ and income inequality among agents thus increases. However, from periods 14 to 20, the skilled agent fraction becomes sufficiently high. Therefore, the ‘skilled cohort effect’ dominates the ‘skill premium effect,’ and income inequality among agents thus decreases. Therefore, we can confirm that the phenomenon called the Kuznets Curve occurs in our example.

Figure 3 shows one more interesting feature of income inequality. We can observe that income inequality again begins to rise in period 20 and increases at a very slow speed for all subsequent periods. In period 20, all agents become skilled. However, from Eq. (4), agents with higher innate potential are more likely to reap the benefits of technological progress. Therefore, as the level of technology increases,

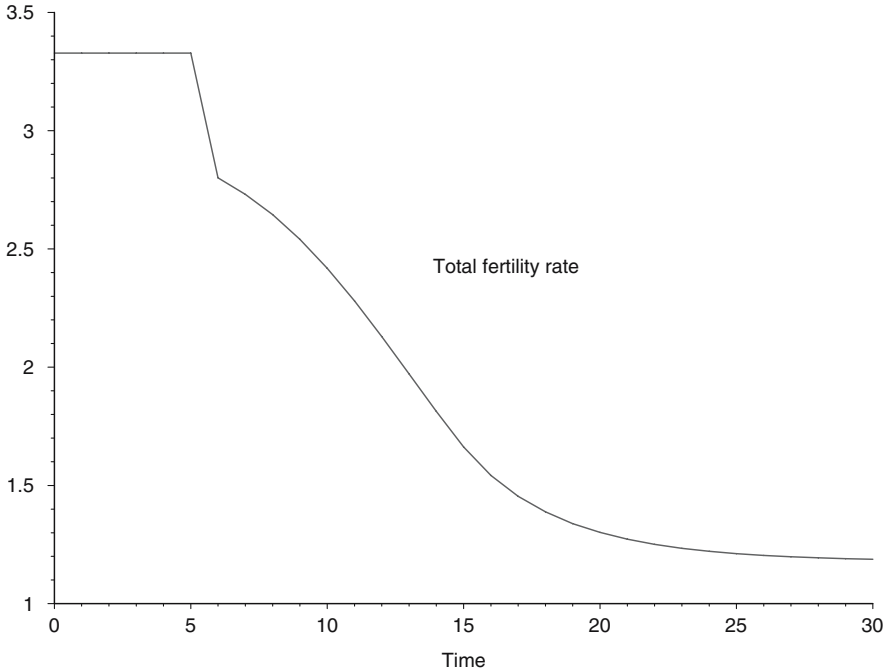


Fig. 5 The evolution of total fertility rate. The educational e decreases from 3 to 2 at period 5

the income inequality *among skilled* agents gradually increases.¹⁷ This gradual increase in income inequality among skilled agents is consistent with recent empirical results described in Atkinson (1995), which demonstrate that income inequality in developed countries such as the USA and the United Kingdom increased in the 1980s and 1990s.

Figures 4, 5 and 6 show that the exogenous decrease in the educational cost also replicates the analogous historically observed pattern of the skilled agent fraction, total fertility rate, and income inequality.

In Figs. 4, 5 and 6, the educational cost decreases at period 5. With this exogenous decline in the educational cost, the economy escapes from the poverty trap, begins to grow, and gradually converges to a fully skilled equilibrium. We can observe that, at period 5, the skilled agent fraction begins to rise and, at period 15, the skilled agent fraction becomes constant at 1: fully skilled equilibrium. With this movement, total fertility rate starts to decrease at period 5. With the rise in skilled agent fraction and technological progress, total fertility rate decreases at subsequent periods, as shown in Fig. 5. In our simulation, as shown in Fig. 6,

¹⁷ However, we can confirm that income inequality in the fully skilled equilibrium is lower than that in the poverty trap equilibrium even if the level of technology approaches infinity.

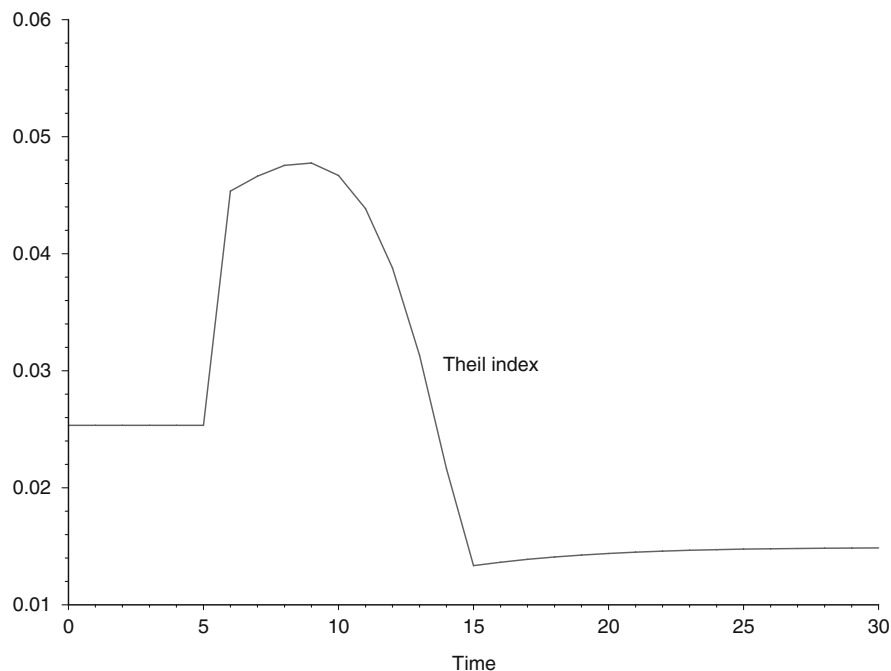


Fig. 6 The evolution of income inequality measured by Theil index. The educational e decreases from 3 to 2 at period 5

from periods 5 to 9, the skilled agent fraction is still sufficiently low. Therefore, the ‘skill premium effect’ dominates the ‘skilled cohort effect’, and income inequality among agents thus increases. However, from periods 9 to 15, the skilled agent fraction becomes sufficiently high. Therefore, the ‘skilled cohort effect’ dominates the ‘skill premium effect’, and income inequality among agents thus decreases. Income inequality again begins to rise in period 15 and increases at a very slow speed for all subsequent periods.

5 Concluding remarks

This paper has constructed an overlapping-generations model with two different types of technology: modern, which can be accessed only by the skilled, and traditional, which can be accessed by the unskilled. With emphasis upon the role of the rise in the skilled wage premium caused by the skill-biased technological change, the model can replicate some observed facts regarding economic development: rises in the fraction of skilled people, declines in fertility, and rises followed by falls in income inequality (the Kuznets Curve). Thus, this paper complements existing theoretical studies which examined the interactions among income, fertility, and income inequality. Furthermore, the model can explain the observed gradual rise in income inequality (among skilled people) in developed countries.

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Appendix A

From Eqs. (9) and (16), the property that $h_i \geq h_{min}$, and the irreversibility of the level of technology A_t , it is easily confirmed that inequality $n_{i,t+1}^s < n^u$ holds for all i , if

$$\psi(A_{t+1})h_{min} + e > w.$$

As the relation $\psi(A_0)h_{min} - Re > w$ holds from Assumption 1, it can be easily demonstrated that the above condition holds.

Appendix B

We define the right-hand and left-hand sides of Eq. (18) as RH and $LH(z_{t+1})$, respectively. Note here that $RH \equiv (1 + R)^{\alpha+\beta+\gamma p} w^{\alpha+\gamma p}$ does not depend on z_{t+1} . Due to inequality $h_i > h_{min}$

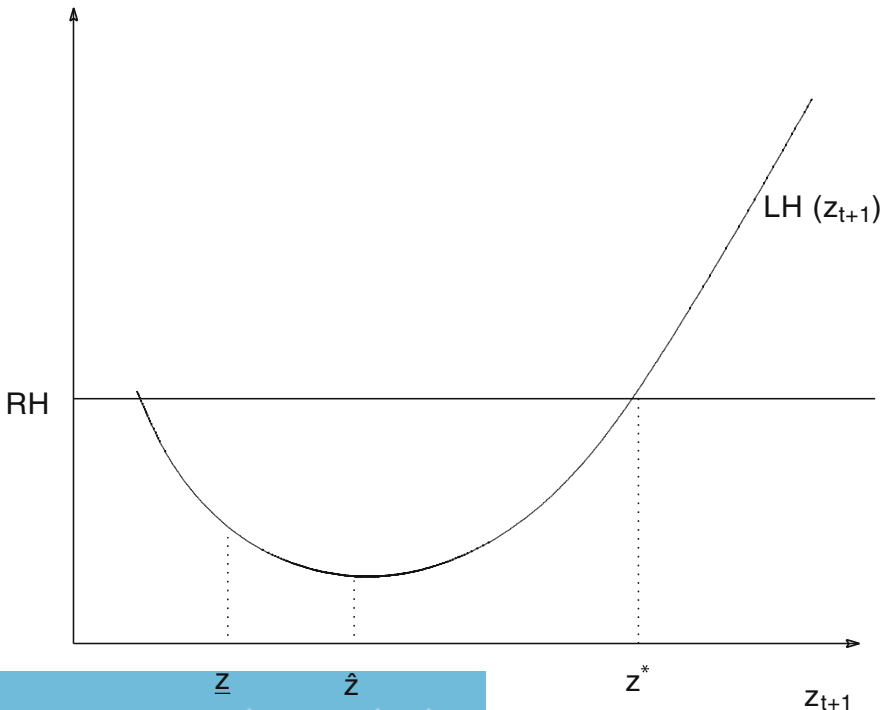


Fig. 7 The existence of z^*

and the irreversibility of the level of technology A_t , it can be easily demonstrated that inequalities $z_{t+1} \geq \underline{z} > 0$ hold for all t . By differentiating $LH(z_{t+1})$ with respect to z_{t+1} , we find

$$LH'(z_{t+1}) = \frac{[z_{t+1} + R(w - e)]^{\alpha + \beta + \gamma p - 1}}{z_{t+1}^{1 + \beta}} [\alpha + \gamma p - \beta R(w - e)],$$

where

$$LH'(z_{t+1}) \begin{cases} > 0 & \text{if } z_{t+1} > \hat{z} \\ \leq 0 & \text{if } z_{t+1} \leq \hat{z}. \end{cases}$$

In this case, \hat{z} is defined as

$$\hat{z} \equiv \frac{\beta R(w - e)}{\alpha + \gamma p}.$$

Therefore, supposing $\underline{z} > \hat{z}$, then $LH(z_{t+1})$ monotonously increases in z_{t+1} for all $z_{t+1} \geq \underline{z}$. On the other hand, let us suppose that $\underline{z} \leq \hat{z}$; then, as shown in Fig. 7, $LH(z_{t+1})$ monotonously decreases in z_{t+1} when $\underline{z} \leq z_{t+1} \leq \hat{z}$, while it monotonously increases in z_{t+1} when $z_{t+1} > \hat{z}$. Note also that $LH(z_{t+1})$ satisfies the condition that $\lim_{z_{t+1} \rightarrow \infty} LH(z_{t+1}) = \infty$.

From Assumption 2, the condition $LH(\underline{z}) < 0$ holds, implying that the graph of $LH(z_{t+1})$ always intersects with the graph of RH only once from below (under inequalities either $\underline{z} > \hat{z}$ or $\underline{z} \leq \hat{z}$). Therefore, there exists a unique z^* that satisfies the following condition:

$$LH(z^*) = RH.$$

Note that the sign of $LH'(z_{t+1})$ evaluated at z^* is always positive. In other words, $LH'(z^*) > 0$.

Agents are assumed to receive education if it is indifferent to them whether they become either skilled or unskilled. Hence, it can be easily shown that the agent whose innate potential h_i is higher than or equal to (lower than) $z^*/\psi(A_{t+1})$ receives (does not receive) education and becomes skilled (unskilled).

Appendix C

In the numerical examples described in Section 4, we specify the three functions $f(h)$, $\psi(A_t)$, and $g(s_t)$ as follows: first, we assume that the innate potential parameter h is uniformly distributed over the interval $[h_{min}, h_{max}]$, implying that the density function $f(h)$ is

$$f(h) = \frac{1}{h_{max} - h_{min}}.$$

Next, the function $\psi(A_t)$, which describes the relationship between the level of technology and the productivity of labor in modern technology, is assumed to take the form:

$$\psi(A_t) = \bar{w}_s \left(\frac{A_t}{1 + A_t} \right)^{1/2}.$$

Therefore, as $A_t \rightarrow \infty$, $\psi(A_t)$ converges to \bar{w}_s . Finally, the function $g(s_t)$, which represents the law of motion of the technological level, is given by

$$g(s_t) = (0.7 + s_t)^{1/2},$$

which, combined with Eq. (19), implies that the economy grows once the skilled agent fraction exceeds 0.3.

In the baseline simulations, the values of parameters are as follows: the utility parameters $\alpha = 0.35$, $\beta = 0.35$, and $\gamma = 0.3$; the cost parameter of rearing a child $\tau = 0.3$; the gross rate

of the returns of storage technology $R = 1.5$; the unskilled wage $w = 5$; the interval of the innate potential parameter $[h_{min}, h_{max}] = [5, 10]$; the productivity parameter $w_s = 5$; the survival probability $p = 0.4$; the tuition fee $e = 3$; and the initial technological level $A_0 = 0.2$.

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